On the Design Method of a Haptic Interface Controller with Virtual Coupling

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Abstract: A haptic interface can be a passive system with virtual coupling as a filter. Virtual coupling has been designed for satisfying passivity. However, it affects transparency of haptic interface as well as stability. This paper suggests new design criterion of a haptic interface controller by considering transparency. As a result, sampling time and the range of impedance or admittance should be considered as well as virtual coupling for desired performance of haptic display. And experiments show that the suggested design criterion can be applied successfully for desired performance.

Keywords: haptic device, haptic interface, virtual coupling

Fig. 1. Interaction between an operator and virtual reality in the ideal case

Fig. 2. Structure of haptic interface and information interaction

2. Haptic Interface for Stability

\[
\begin{bmatrix}
F_h \\
-v_{de}^\ast
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix} \begin{bmatrix}
v_h \\
F_{de}^\ast
\end{bmatrix}
\]

\[Z_h = \frac{F_h}{v_h} = \frac{F_{de}^\ast}{v_{de}^\ast} = Z_e \]

A haptic system consists of an operator, virtual reality, and a haptic interface. In Figure 1, impedance control at an operator side and admittance control at virtual reality are applied. An operator commands \(v_h\) and \(F_{de}^\ast\) to virtual reality. Virtual reality accepts it as desired velocity, \(v_{de}^\ast\). Virtual reality returns reaction force, \(F_{de}^\ast\), and an operator feels \(F_h\). Eq.(1) represents an ideal haptic system as shown in Figure 1. In Eq.(1), the commanded velocity and reaction force are transmitted perfectly and the impedance of an operator, \(Z_h\), is coincident with the impedance of virtual reality, \(Z_e\).

However, an ideal haptic system can not be realized since information is always distorted between continuous time and discrete time domain. In other words, the information distortion is inevitable because of digitizer and zero order hold which connect the two sides. Virtual coupling is needed in order to overcome the problem. Figure 2 shows impedance haptic display. \(F_h\) and \(v_h\) mean reaction force from haptic device and velocity commanded from an operator. \(v_{de}^\ast\) and \(F_{de}^\ast\) are desired velocity transmitted to virtual reality and desired reaction force to an operator. Asterisk means discrete value. \(v_{de}^\ast\) is discrete value of \(v_{de}\), through digitizer and \(F_{de}^\ast\) is continuous value of \(F_{de}^\ast\) through zero order hold, ZOH. Virtual coupling makes desired velocity to virtual reality, \(v_{de}^\ast\), and desired force to display, \(F_{de}^\ast\) with \(F_{de}^\ast\) and \(F_{de}^\ast\).

2.1. Virtual Coupling for Admittance Display

Admittance haptic display means that an operator commands force to virtual reality and velocity of virtual reality returns through a haptic device. Though
In Figure 3, if there is no virtual coupling, means sampling time. ZOH is zero order hold which a haptic device, a controller is needed. K_Pi means PD controller. By using the controller, a haptic device can be in desired position.

\[
K_{Pi}(z) = \frac{K_p}{s} + K_d\frac{(z-1)}{T_z}
\]

To guarantee passivity of Eq.(3),

\[
Re\left(C_{A1}(\omega)\right) \geq 0
\]

\[
Re\left(\frac{\text{ZOH}(\omega)G(z)C_{A1}(\omega)}{Z(z) + K_{Pi}(\omega)}\right) \geq 0
\]

\[
2Re(C_{A1}(\omega))Re\left(\frac{\text{ZOH}(\omega)G(z)C_{A1}(\omega)}{Z(z) + K_{Pi}(\omega)}\right) \geq |C_{A2}C_{A3}\text{ZOH}(\omega)G(z)Y_e|^2 \forall \omega \geq 0
\]

Stability of the suggested admittance haptic display is guaranteed by how to design virtual coupling factors, C_{A1}(\omega), C_{A2}(\omega), C_{A3}(\omega) and C_{A4}(\omega). Virtual coupling has been designed in order to satisfy only passivity in many papers. However, this equation is only for stability. Transparency as well as stability should be considered for haptic display.

3. New Virtual Coupling Design of Admittance Display for transparency

In this section, design method of virtual coupling is considered to guarantee transparency as well as stability. From Eq.(10), following shows the relation between admittances of an operator and virtual reality.

\[
Y_h = \frac{v_h}{F_h} = \frac{ZD(z) + K_{Pi}(z)}{1 + C_{A4}Y_e}
\]

\[
ZD(z) + K_{Pi}(z)
\]

\[
1 + C_{A4}Y_e
\]

Y_h and Y_e mean admittance of an operator and virtual reality. Eq.(14), the first part of right side is determined if haptic device, PD controller, and virtual coupling factors are given. The second part of right side is dependent on admittance of virtual reality, Y_e. Therefore, the second part decides transparency with virtual coupling factor.

\[
D(z) \propto Y_h - \frac{ZD(z) + K_{Pi}(z)}{1 + C_{A4}Y_e}
\]

\[
D(z) = \frac{C_{A2}C_{A3}\text{ZOH}(\omega)G(z)Y_e}{1 + C_{A4}Y_e}
\]

\[
D(z) = -\frac{(z+1)a_{A1}(z)b_{A2}(z)b_{A3}(z)G(z)Y_e}{2z(a_{A1}(z) + b_{A1}(z)Y_e)a_{A2}(z)a_{A3}(z)}
\]

Following condition should be satisfied to guarantee the passivity of Eq.(10).

\[
Re(C_{A1}(\omega)) \geq 0
\]

\[
Re\left(\frac{\text{ZOH}(\omega)G(z)C_{A1}(\omega)}{Z(z) + K_{Pi}(\omega)}\right) \geq 0
\]

\[
2Re(C_{A1}(\omega))Re\left(\frac{\text{ZOH}(\omega)G(z)C_{A1}(\omega)}{Z(z) + K_{Pi}(\omega)}\right) \geq |C_{A2}C_{A3}\text{ZOH}(\omega)G(z)Y_e|^2 \forall \omega \geq 0
\]
From Eq.(17), poles of $D(z)$ are on points which satisfy that
\[ z = 0, \quad a_{A1}(z) + b_{A1}(z)Y_e = 0, \]
\[ a_{A2}(z) = 0, \quad a_{A3}(z) = 0, \quad \text{and} \quad G(z) = 0. \]
Therefore, $D(z)$ has the best performance when $a_{A2}(z)$ and $a_{A3}(z)$ are real constants. $C_{A2}(z)$ and $C_{A3}(z)$ should be real constants for causality. Besides, $C_{A2}(z)$ and $C_{A3}(z)$ are 1 and $-1$ in order for coincidence of signs among $F_{de}, v_{de}, F_{dh},$ and $v_{dh}$.

\[ D(z) = \frac{(z + 1)G(z)Y_e}{2(1 + C_{A1}(z)Y_e)} \]  

(18)

Eq.(18) should satisfy the followings for stable haptic interface.

\[ \text{Re}(C_{A1}(z)) \geq 0 \]  

(19)

\[ \text{Re}(ZOH(z)G(z)C_{A4}(z) + \frac{1}{z + 1}) \geq 0 \]  

(20)

\[ 2\text{Re}(C_{A1}(z))\text{Re}(ZOH(z)G(z)C_{A4}(z) + \frac{1}{z + 1}) \geq \left| 1 - \frac{ZOH(z)Y_e}{2} \right|^2 \]  

(21)

\[ \text{Re}(C_{A1}(z)) \geq 0 \]  

(22)

A passive system as shown in Figure 5 is considered in order to satisfy Eq.(19).

\[ (F_{dh} - F_{de} - m_2v_{de}) \left( \frac{1}{z + 1} + \frac{1}{c_2} + \frac{1}{m_2s} \right) = v_{de} \]  

(23)

\[ F_{de} = (v_{de}^* - v_{de}) \left( \frac{k_2}{2} + c_1 \right) \]  

(24)

Let $m_1 = 0$ since $C_{A2}(z) = 1$ and $C_{A3}(z) = -1$. Besides, if $k_2 = 0$ in which $F_{dh}$ is not fluctuated when $F_{dh} = 0,$

\[ F_{de} = F_{de}^* - Z_{Ave}(z)v_{de}^* \]  

(25)

\[ v_{de} = v_{de}^* + \frac{1}{Z_{Ave}^*}F_{de} \]  

(26)

where

\[ Z_{Ave}(z) = \frac{1}{Y_{Ave}(z)} \]  

(27)

\[ Y_{Ave}(z) = \frac{1}{c_2} + \frac{1}{m_2s}|z^{m(s-1)}/Tz| \]  

\[ Z_{Ave}(z)' = \frac{k_1}{s} + c_1|z^{m(s-1)}/Tz| \]  

From Eq.(25) and Eq.(26),

\[ C_{A1} = Z_{Ave}(z) \]  

(28)

\[ C_{A4} = \frac{1}{Z_{Ave}^*(z)} \]  

(29)

**Assumption 1**: A haptic device is so stiff that it cannot be controlled by an operator.

**Assumption 2**: Control frequency is high enough and PD controller is optimized.

Now, Eq.(20) is always satisfied. Stability and transparency can be determined by followings with Assumption 1 and Assumption 2 by which $G(z) \rightarrow 1$ and $\frac{1}{Z_{Ave}(z) + K_{Pr}/(z)} \rightarrow 0$.

\[ 2\text{Re}(Z_{Ave}(z))\text{Re}(ZOH(z)Y_e) \geq \left| 1 - \frac{ZOH(z)Y_e}{2} \right|^2 \]  

(30)

\[ D(z) = \frac{(z + 1)Y_e}{2z(1 + Z_{Ave}(z)Y_e)} \]  

(31)

From Eq.(31), DC-gain of $D(z)$ is $Y_e$ and convergence rate is determined by $(1/c_2 + Y_e)/(1/c_2 + T/m_2 + Y_e)$. It means that a haptic interface is affected by admittance of virtual reality, $Y_e,$ and sampling time, $T,$ as well as virtual coupling factors. In other words, there are limits of expressible admittance and increasing sampling time does not mean better performance without exception.

The relation among sampling time, the range of expressible admittance, and virtual coupling factors will be explained. Though admittance of virtual reality changes from 0 to infinity randomly, admittance is assumed to be fixed to clarify the relation. Eq.(31) can be derived as the following.

\[ D[n] = \frac{1}{c_2 + Y_e + T/m_2}D[n - 1] \]  

(32)

\[ D[n] = Y_e \left( 1 - \left( \frac{1/c_2 + Y_e}{1/c_2 + T/m_2} \right)^n \right) \]  

(33)

Let $Q_A = \frac{T/m_2}{1/c_2 + Y_e}$

$NT$ means the time in which $D(z)$ reaches $(1 - \lambda')Y_e$ from 0 when admittance of virtual reality is $Y_e$.

For example, if $D(z)$ reaches $95\%$ of $Y_e$ in 0.1 (sec) and sampling time is 0.01, $N = 10$ and $\lambda' = 0.05.$

\[ \left( \frac{1}{1 + Q_A} \right)^N \leq \lambda' \]  

(34)

\[ Q_A \geq \lambda' \frac{1}{N} - 1 \]  

(35)
Fig. 6. $c_2 = 100$, $m_2 = 0.01$, and $T = 0.005$

Q_A has lower limit when $\mathcal{N}$ and $\mathcal{X}$ are determined in Eq.(33).

$$\mathcal{N} \geq \frac{\ln \mathcal{X}}{\ln (1/(1 + Q_A))} \quad (34)$$

Eq.(34) shows that more than $\mathcal{N}$ is needed to converge $(1 - \mathcal{X})Y_e$ when virtual coupling factors are determined.

$$T/m_2 \geq (1/c_2 + Y_e) \left( \mathcal{X}^{-N} - 1 \right) \quad (35)$$

Virtual coupling factor, $m_2$, and sampling time, $T$ should be determined as shown in Eq.(35) to converge $(1 - \mathcal{X})Y_e$ in $\mathcal{N}$ step for given $c_2$ and $Y_e$.

$$Y_e \leq \left[ T/m_2 - \left( \mathcal{X}^{-N} - 1 \right) /c_2 \right] / \left( \mathcal{X}^{-N} - 1 \right) \quad (36)$$

If virtual coupling factors and sampling time are given, $Y_e$ should satisfy Eq.(36) in order to converge $(1 - \mathcal{X})Y_e$ in $\mathcal{N}$ step. Eq.(36) represents that there is limit of expressible admittance when other factors are given.

4. Experiment for Admittance Display

In this section, the results stated above are confirmed through experiments, qualitatively. Only admittance display is considered because of limit of page.

In the simulations, it is assumed that admittance is constant in order to analyze easily and to prove relation among factors which consist of haptic interface. Though admittance changes from 0 to infinity, fluctuation of it is so small in short sampling time that relation mentioned above can be shown in real haptic display. For example, it is obvious that a haptic interface has upper limit of admittance to display for the selected sampling time and virtual coupling factors. In other words, sampling time and virtual coupling factors should be changed according to inequalities mentioned above to express desired admittance.

Figure 6-10 show admittance of virtual reality and an operator when $m_2$ is changed from 0.01 to 100, $c_2 = 100$, and $T = 0.005$. Circles and crosses represent admittance of an operator and virtual reality, each other. From Eq.(35)-Eq.(36), it can be expected that increasing $m_2$ affects transparency negatively. Figure 6 shows that admittance of an operator is coincident with that of virtual reality. However, the coincidence is almost broken especially in high admittance as shown in Figure 7 and 8. Finally, Figure 9 and Figure 10 show that transparency is not guaranteed anymore.

References